Image Denoising using M-Band Ridgelet Transform

Ramanjyot Kaur* and Palvinder Singh Mann**

* Department of Computer Science & Engineering, SBBSIET
** Department of Computer Science & Engineering, DAVIET Jalandhar, Punjab, India.

(Received 13 July 2013 Accepted 23 July 2013)

Abstract- In this paper, a novel image denoising algorithm using M-band ridgelet transform is proposed for image denoising. The performance of the proposed method is tested on ultrasound images which are corrupted with Gaussian noise. The performance of the proposed method is compared with the existing ridgelet and curvelet transform in terms of peak-signal to noise ratio (PSNR) and mean square error (MSE). The results after investigation of proposed method show a significant improvement in terms of PSNR and MSE as compared to other existing denoising algorithms on ultrasound images.

Keywords-Image denoising, M-Band Ridgelet Transform, Curvelet, PSNR, MSE

1. INTRODUCTION

Noise is added in the digital images during image acquisition and transmission. Images are corrupted during transmission mainly due to interference in the channel used for transmission [1]. Noise arises as a result of unmodelled processes going on in the production and capture of the real signal. It is not part of the ideal signal and may be caused by a wide range of sources, e.g., variations in the detector sensitivity, environmental variations, the discrete nature of radiation, transmission or quantization errors, etc. The characteristics of noise depend on its source, as does the operator which best reduces its effects [9].

2. TYPES OF NOISE

Noise is a very common problem in the digital images. An image gets corrupted with different types of noise [7] during the processes of acquisition, transmission/reception, and storage/retrieval. Noise may be classified as substitutive noise (impulsive noise: e.g., salt & pepper noise, random-valued impulse noise, etc.), additive noise (e.g., additive white Gaussian noise) [8] and multiplicative noise (e.g., Speckle noise) [14].

2.1. Salt & peppers’ noise

Salt and pepper noise is a form of noise typically seen on images. It represents itself as randomly occurring white and black pixels [2]. An effective noise reduction method for this type of noise involves the usage of a median filter or a contra harmonic mean filter. Salt and pepper noise creeps into images in situations where quick transients, such as faulty switching, take place [10].

2.2. Gaussian noise

Gaussian noise can be analytically described and has a characteristic bell shape [12]. With uniform distribution, the gray level values of the noise are evenly distributed across a specific range. Gaussian noise is statistical noise that has its probability density function equal to that of the normal distribution, which is also known as the Gaussian distribution. In other words, the values that the noise can take on are Gaussian-distributed. A special case is white Gaussian noise [11], in which the values at any pairs of times are statistically independent (and uncorrelated).

2.3. Speckle noise

Speckle noise is a form of multiplicative locally correlated noise which occurs in imaging applications such as medical ultrasound image interpretation [13]. One of the fundamental problem of ultrasound images is the poor quality, caused mainly by
multiplicative speckle noise. Speckle is mainly caused by interference between coherent waves which are backscattered by targeted surfaces and arrives out of phase at the sensor [5]. Speckle can be modeled as random noise (irregular pattern), which degrades the detection of low contrast lesions and also reduces the ability of a human observer to resolve fine detail [16].

3. RIDGELET TRANSFORM

3.1. Radon Transform

The Radon transform [5] of an object \( f \) is the collection of line integrals indexed by \( (\theta, t) \in [0, 2\pi) \times \mathbb{R} \) given by

\[
Rf(\theta, t) = \int f(x_1, x_2)\delta(x_1 \cos \theta + x_2 \sin \theta - t) \, dx_1 \, dx_2
\]

where \( \delta \) is the Dirac distribution. The ridgelet coefficients \( CRT_{(a,b,\theta)} \) of an object \( f \) are given by analysis of the Radon transform via

\[
CRT_{(a,b,\theta)} = \int Rf(\theta, t) a^{-1/2} \psi((t-b)/a) \, dt
\]

3.2. Discrete Ridgelet Transform (DRT)

A continuous ridgelet transform [15] is calculated by applying 1D wavelet transform to the slices of radon transform \( R_j(\theta, \cdot) \). In radon transform a famous projection-slice theorem is used

\[
\hat{f}(\omega \cos \theta, \omega \sin \theta) = \int R_j(\theta, t) e^{-2\pi i \omega t} \, dt
\]

This theorem says that the Radon transform can be obtained by applying the one-dimensional inverse Fourier transform to the two-dimensional Fourier transform of function restricted to radial lines through the origin. To complete the ridgelet transform, apply a one-dimensional wavelet transform along the radial variable in Radon space.

3.3. M-Band Ridgelet Transform (MRT)

M-Band ridgelets [7] based on the ridgelet transform combined with a M-band wavelets is proposed in this paper for image denoising.

Figure 1 illustrates the flowchart for the digital implementation of proposed M-Band ridgelet transform and the algorithm for the image denoising using M-band ridgelet transform is given as follows.

Algorithm:
Input: Noise Image; Output: Denoised Image
1. Load the image and convert it into gray scale (if it is RGB).
2. Apply the 2D-FFT and partition into slices.
3. Apply the 1D-FFT\(^1\) on each slice.
4. Apply the 1D-M-band wavelets on each slice.
5. Get the M-Band ridgelet responses.

---

![Flowchart of Ridgelet and M-Band Ridgelet Transform](image-url)
4. PROPOSED DENOISING ALGORITHM

Algorithm:
1. Apply M-band ridgelet transform to the noisy image and get the scaling coefficients and M-band ridgelet coefficients.
2. Chose the threshold by neighCoeff thresholding and apply thresholding to the M-band ridgelet coefficients.
3. Reconstruct the M-Band Ridgelet coefficients thresholded and get the denoised image.

5. EXPERIMENTAL RESULTS AND DISCUSSIONS

Removal of noises from the images is a critical issue in the field of digital image processing. The phrase peak signal to noise ratio, often abbreviated PSNR, is an engineering term for the ratio between the maximum possible power of a signal and the power of corrupted noise that affects the fidelity of its representation.

$$PSNR = 20 \log_{10} \left( \frac{255}{MSE} \right)$$

$$MSE = \frac{1}{mn} \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} \| I(i, j) - k(i, j) \|^2$$

Figure 2 and Table I illustrate the denoising results of various methods on three ultrasound images[3] under Gaussian noise with zero mean and 0.1 variance.

Figure 2: Results of denoising methods under Gaussian noise with zero mean and 0.1 variance.
### Table I. Denoising Results of Various Method Under Gaussian Noise with Zero Mean and 0.1 Variance

<table>
<thead>
<tr>
<th>Images</th>
<th>White Gaussian noise with zero mean and 0.1 variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ridgelet</td>
</tr>
<tr>
<td></td>
<td>PSNR</td>
</tr>
<tr>
<td>US1</td>
<td>15.49</td>
</tr>
<tr>
<td>US2</td>
<td>15.43</td>
</tr>
<tr>
<td>US3</td>
<td>14.93</td>
</tr>
</tbody>
</table>

### 6. Conclusion

The ridgelet transform with M-band wavelet transform called M-band ridgelet transform is proposed in this paper for medical image segmentation. The performance of the proposed method is tested on ultrasound images under Gaussian noise. The results of the proposed method is compared with the ridgelet and curvelet transform in terms of PSNR, MSE. The results after being investigated shows a significant improvements compared to the ridgelet and curvelet denoising algorithms.

### References


